Dynamic Airline Scheduling

- Models, Algorithms, and Experiments

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Outline

• Introduction to the problem
• Dynamic airline scheduling
  – The concept
  – Modeling architecture
  – Mathematical models
• Experimental evaluation
• Conclusions
Introduction

• It has been a major challenge for the airline industry to design the flight schedule and fleeting to match fluctuating passenger demand.

• Sources of demand variation
  – Seasonality
  – Day of week
  – Time of day
  – Response to competition
  – Inherent random fluctuation
Current Practice

• Most airlines anticipate the schedule and fleeting remain unchanged till departure
  – Flight schedule is typically generated 6 months to 1 year before departure. Fleeting is established about 2 months in advance
• Revenue management systems controls the interaction between supply and demand
• RM also helps to smooth demand variation
• However, we still observe flight departures with empty seats or spills

- Empty seats
- Wasted resources
- Excessive operating cost

- Passenger spill
- Lost revenue
Dynamic Airline Scheduling

• There is a need to develop new decision tools to manage demand variation

• The idea of dynamic airline scheduling
  – Re-time flights within a small time window and swap aircraft within the same family in the booking process to shift capacity to where is needed
  – Such re-optimization is performed for each individual departure date
Illustration of the Idea: Swap

Planned Schedule and Fleeting

Day 1

Day 2

Day 2 with new fleeting

Swap
No spill
Illustration of the Idea: Re-time

Planned Schedule and fleeting

Day 1

Day 2

Day 2 after retime

Less than MinCT

10 more demand

Become a feasible connection

Spill at least 6
Re-time Increases Swap Opportunities

Planned Schedule and fleeting

A320

A319

A319

Cannot Swap

New Schedule and fleeting

A320

A319

A320
Where Are Swap/Retime Opportunities?

- Airlines typically operate on a hub-and-spoke network with banked schedules

- Hub-and-spoke network provides plenty of swap opportunities

- In a perfectly banked schedule, re-time flights does not generate much interest
Where Are Swap/Retime Opportunities?

- A recent trend to de-peak the schedule at hub

- Flight re-timing becomes attractive
Make Use of Improved Demand Data

- Make wiser decision at time of re-optimization
  - Information from revealed bookings
  - Reduced final demand forecast error
When to re-optimize?

• If too far from departure date
  – Little information about revealed bookings
  – Poor forecast quality
  – More flexibility in making schedule and fleeting changes

• If too close to departure date
  – A lot of information about the past
  – Good forecast quality
  – Little flexibility in making schedule and fleeting changes

• Make a balance

• Re-optimization frequency

• In our experimental studies
  – One-time re-optimization at 21 days prior departure
Model Architecture
The Base Case

Period 1 pax demand → Passenger Mix Model → Period 1 Pax Assignment → Remaining Leg capacity → Passenger Mix Model → Output

Booking Limit

21 day Prior departure

Period 2 pax demand
The Dynamic Case

New schedule guarantees:
• All connecting itineraries sold in Period 1 remain feasible
• # of aircraft for each fleet overnighted at each station is no more than originally planned

Period 2 Pax Demand Forecast

Re-optimize fleet & flight timing

New schedule

# of Aircraft Overnighted At Each Station For Each Fleet

Itineraries to be Preserved in Period 2

Seats Taken On Each Leg

Passenger Mix Model

Period 1 Pax Demand

Period 1 Pax Assignment

Remaining Leg capacity

Passenger Mix Model

Output

Period 2 Pax demand

Booking Limit

21 day Prior departure

Departure date
Re-optimization Formulation (1)

- IFAM + time windows + additional side constraints
Re-optimization Formulation (2)

\[ \sum_{g \in G} y_g \zeta_i = y_g^0, \forall i \in STATION, \forall \pi \in \Pi \]

\[ \sum_{l \in L} \sum_{k \in C(l)} \sum_{\pi \in \Pi} \gamma_{l,k_\pi}^a f_{l,k_\pi} \leq MAX_{ACT}^a, \forall i \in T \]

\[ \sum_{l \in L} \sum_{k \in C(l)} \sum_{\pi \in \Pi} \gamma_{l,k_\pi}^d f_{l,k_\pi} \leq MAX_{ACT}^d, \forall i \in T \]

\[ \sum_{\pi \in \Pi} f_{l_1,k_1\pi} + \sum_{\pi \in \Pi} f_{l_2,k_2\pi} \leq 1, \forall (l_1, l_2) \in P, (k_1, k_2) \notin \]  

\[ f_{l,k_\pi} \in \{0, 1\}, \forall l \in L, k \in C(l), \pi \in \Pi \]

\[ y_g \pi \geq 0, \forall g \in G, \pi \in \Pi \]

\[ x_{m,r} \geq 0, \forall m \in M, r \in R(m) \]

\[ z_{\pi} \geq 0, \forall \pi \in \Pi \]
Solution Technique

• Branching Strategy
  – Branching on a single fractional $f_{lk\pi}$ is not effective
  – Define Special Ordered Sets (SOS) for the flight cover constraints using capacity as weight.
    \[
    \sum_{k \in C(l)} \sum_{\pi \in \Pi} f_{lk\pi} = 1
    \]
  – Split into two sets while branching
    \[
    \sum_{(k,\pi) \in S} f_{lk\pi} = 1 \quad \text{or} \quad \sum_{(k,\pi) \in \bar{S}} f_{lk\pi} = 1
    \]
  – Result in a more balanced division and improved run-time performance

• Column Generation
  – When flight copies are generated in a time window, the number of possible passenger itineraries increases substantially
Experimental Evaluation
Experiment Setup

• Airline statistics
  – Daily flights: 832
  – Daily passengers: 50k path pax / 70k leg pax
  – 302 inbound and 302 outbound flights at hub daily

• Retime window
  – +/- 15 minutes

• Refleet
  – A320 & A319
  – CRJ & CR9

• We experimented with 7 days in a week

• Two scenarios about forecast quality
  – Perfect forecast
  – Historical average
Characterize Forecast Quality

- Scatter plot visualizes forecast quality
- Define relative deviation:

\[ RD_m = \left| \frac{D_{\text{forecast}} - D_{\text{actual}}}{D_{\text{actual}}} \right| \]
Improvement In Profitability

- Consistent improvement in profitability
  - Perfect information
    - 4-8% improvement in profit
    - 60-150k daily
  - Average information
    - 1-5% improvement in profit
    - 30-80k daily
- System load factors up 0.5 ~1%
- Benefit remains significant when using historical average as forecast
Comparison between Retime and Swap

- $P^{\text{(swap+retime)}} \geq P^{\text{(swap)}} + P^{\text{(retime)}}$
- Improvement of retiming exceeds that of swapping in most cases
- Retiming always improves profit
- Swapping is more sensitive to forecast quality and can have negative results
- $P^{\text{(retime)}} > P^{\text{(swap)}}$ in most cases
Increases in Passengers

- Retiming effectively captures connecting passengers
Properties of New Itineraries

• What is the quality of new itineraries?
• Define two types of new itineraries

- Average connection time for new itineraries
  - 35 minutes for Type 1 and 170 minutes for Type 2
- The majority of passengers on new itineraries are Type I (80%)
Properties of New Itineraries

![Graph showing the percentage of Type I and II passengers and the average connection time over the days of August.](image)
Properties of Re-optimization Decision

- Re-fleeting: ~ 100 fleeting changes
- Re-timing: ~ average retime of 8 minutes
- What if we restrict the number of re-timed flights?

- 50% of the full benefit was achieved when 12% flights are retimed
- 70% of the full benefit was achieved when 25% flights are retimed
Re-timing Decision Analysis

- Are there inefficiencies in the base schedule?
- Did we make consistent re-timing on some flights?

- 95% of the flights are rarely retimed
- 2% of the flights are frequently retimed and in the same direction
- Allowing 2% of flights to be retimed yields little improvement
Review of the Approach

• Maintenance
  – Due to lack of data, maintenance considerations are ignored

• Restriction on the number of overnight aircraft for each fleet at each station
  – Perform joint optimization of consecutive dates
  – For intermediate dates, this constraint can be relaxed

• Passenger assignment module
  – We use a Passenger Mix Model with recapture
  – Simulation based approach
Summary

- Introduced a new mechanism in dynamic scheduling - flight retiming
- Developed models that integrate both retiming and swapping
- Demonstrated significant improvements in profitability using data from a major airline
- Studied and compared the effectiveness of retiming and swapping
  - When applied alone, retiming has larger contribution toward profitability than swap
  - Retiming is less sensitive to forecast quality
Acknowledgement

• Sloan Foundation through the MIT Global Airline Industry Program
Question?
References


Model for Period 1

\[
\begin{align*}
\text{maximize} & \quad \text{Maximize Profit} \\
\text{subject to} & \quad \text{Serve Pax} \\
& \quad \text{Capacity Constraints} \\
& \quad x_{mr} \geq 0, \forall m \in M, r \in R(m)
\end{align*}
\]
### Model for Period 2 (static)

<table>
<thead>
<tr>
<th>maximize</th>
<th>Maximize Profit</th>
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<tbody>
<tr>
<td>subject to</td>
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<tr>
<td>Serve Pax</td>
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<tr>
<td>Capacity Constraints</td>
<td></td>
</tr>
<tr>
<td>$x_{mr} \geq 0, \forall m \in M, r \in R(m)$</td>
<td></td>
</tr>
</tbody>
</table>
Model for Period 2
(Dynamic Using New Schedule)

\[
\begin{align*}
\text{maximize} & \quad \text{Maximize Profit} \\
\text{subject to} & \quad \text{Serve Pax} \\
& \quad \text{Capacity Constraints} \\
& \quad x_{m,r} \geq 0, \forall m \in M, r \in R(m)
\end{align*}
\]